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REVIEWS

Collected Works of R. T. Jones. NASA, 1976. 1025 pp. $15.25.

1. Introduction

To celebrate the 65th birthday of one of the world's greatest aerodynamicists, Robert T. Jones, NASA has published this volume of his Collected Works. Everyone concerned with aerodynamics, that is, with the air forces acting on aircraft and with the resulting aircraft dynamics, will wish to possess this collection of classic papers in the field offered at such a reasonable price by Dr Jones's proud employers. Fundamental ideas are often best understood by reading the papers where they were first proposed, and this is doubly the case when an idea's originator has Bob Jones's clarity, simplicity and penetration of writing style. Sweptback wings brought all the world far closer together; here, you can see the essential ideas behind them in the process of formation.

An inspired choice for the writer of the biographical Introduction (Professor W. L. Sears) has added greatly to the pleasure of those who will use this book not only to understand aerodynamics better, but also to appreciate men of genius in general and a particular man of genius who is dear to all working in his field. Reading the collected papers in the light of this illuminating biographical sketch by Bill Sears is the perfect way to come to a fuller understanding of the different factors that influenced the development and flowering of Bob Jones's special personality and unique talents.

Those buying this book will already possess the 240-page *Aerodynamics of Wings at High Speeds* by R. T. Jones and Doris Cohen, constituting volume VII, Section A, of the Princeton series *High Speed Aerodynamics and Jet Propulsion*, and published in 1957. It is naturally important to take into account that masterly review, the principal omission from these Collected Works, in order to be able to trace clearly the full development of Dr Jones's aerodynamic ideas during the later years of his remarkable career.

The degree, of course, is an honorary one: Bob Jones's mind worked much too fast to accommodate the pace of courses at his University and after only a year there he left to go straight into the aircraft industry. A highly professional paper doing the stress analysis of the Pobjoy Raceplane's wing, written for the Nicholas–Beasley Airplane Co. in 1929, opens this Collected Works and establishes what a sound foundation on the structural side of aeronautics Bob Jones had acquired, even though all his later aeronautical activity was to be in aerodynamics. Interestingly, even his later papers on aircraft design problems in general are concerned almost exclusively with aerodynamic aspects; structural problems appear again only in the very last of these, a Boeing report jointly written with J. W. Nisbet which develops an elegant three-degree-of-freedom model for the aeroelastic characteristics of a yawed-wing aircraft.

When the depression put Nicholas–Beasley and many other firms out of business, a position as liftboy was much appreciated; particularly as the strategically placed lift (opposite the Library of Congress) brought Bob in contact at last with an aerodynamics genius, Dr Max M. Munk, fitted to be the teacher of a man of genius. The whole aerodynamic career of R. T. Jones was profoundly influenced by that teaching. Further contacts made from the same stragetically placed lift won him his first post with NACA (as it then was) in Langley Field....

In the nineteen-thirties the great aerodynamic problems were in the sphere of aircraft dynamics, including stability, gust response, controllability and handling. Between 1936 and 1944 a series of classic papers in these fields by R. T. Jones appeared which are worth analysing in a separate part (§ 2) of this review; evidently, the exceptional fame and importance of the later work on steady-flow forces (§ 3) must not be allowed to dim the author's earlier aircraft-dynamics achievements.

Outside the subject-matter of these fields reviewed in detail in §§ 2 and 3 below, the Collected Works include material within four other areas, of which three are potentially of concern to readers of the *Journal of Fluid Mechanics* (the fourth being telescope optics). The last article, on an interesting topic in violin acoustics, was published in the Newsletter of the Catgut Acoustical Society. Articles on pages 927–956 apply the ideas of supersonic aerodynamics to problems of accelerated motion, treated according to the Special Theory of Relativity. Dr Jones here points out the advantage of using what in supersonic aerodynamics are called characteristic co-ordinates.

A still more substantial interest of some of the more recent years of Bob Jones's life, when he temporarily left NASA to work at Avco–Everett Research Laboratory, was in the dynamics of the cardiovascular system, including the possible effectiveness of some of the heart-assist devices on which that Laboratory was then working. In this area Dr Jones in his late fifties demonstrated his continuing capacity for rapidly and effectively reading himself into a new field. Volume 1 of *Annual Review of Fluid Mechanics* (1969) included a masterly review article by him on blood flow, while heart-assist devices in general were thoroughly reviewed by him at the important 1970 Biomechanics meeting in La Jolla. Much of the typical R. T. Jones insight was poured into this later work (here reproduced); as, for example, in the elegant use of simple velocity-potential distributions to model the flow pattern associated with ejection from the heart itself. Haemodynamics is the richer for having been one of the subjects which he touched.

Aircraft, however, had captured Bob Jones's imagination in his teens and, perhaps in consequence of that, became the research field matched most perfectly to his unique powers. He is assured permanently of one of the great places in the history of aeronautical science, and the rest of this review scans through the aerodynamic parts of his Collected Works, to suggest why that is.

2. Aircraft dynamics

By 1936, when Jones wrote his first papers on aircraft dynamics, the mathematical theory determining conditions for aircraft motions to be stable was already far advanced, and quite intricate, but Jones's self-taught mastery of mathematics was shown by his rapid unravelling of the various 'tricks', commonplace now but not then, for arriving with adequate accuracy at a simplified view of its complexities. That mastery was shown also when he immediately went beyond the stability theory proper to calculate an aircraft's transient response to gusts. Here, he used his complete grasp of the technique by which Heaviside's operational calculus (now usually called Laplace-transform theory) generates forced-motion solutions to dynamic problems whose free motions are known through the Carson Integral (now usually referred to as the convolution integral or the Duhamel superposition theorem) and the Bromwich expansion of the inverse Laplace transform.

Soon, Jones was using the same ideas for the quantitative estimation of controllability: determination of the magnitude of control moments needed to generate a turn or to bring a disturbance under control. All this work gave him a properly cautious attitude to stability theory: he gave one of the first clear quantitative statements of that conflict between stability and controllability which had been instinctively known to bicycle experts like the Wright brothers, while emphasizing the need for adequate stability margins, without which even a theoretically stable aircraft could be excited into uncomfortable oscillations.

Soon the mathematics had been developed far enough for an aircraft design's controllability to be predicted from wind-tunnel measurements (1936 paper with F. E. Weick). Excellent agreement with subsequent flight tests was obtained. Lateral control was Jones's special study, and he particularly stressed how at high lift coefficients damping in roll was at risk because the descending wing could stall, with the much feared spin (an exponential divergence predicted if $L_p N_r < L_r N_p$) as one possible consequence.

During the next two years he perfected methods of calculating the contributions to all significant lateral-stability derivatives from aircraft wings of various taper, twist and dihedral. In these intricate calculations from lifting-line theory we can discern already the future matter of wing airflow analysis. They meant that the triple weapon of aerodynamic calculation, wind-tunnel measurement and flight test was available for advancing the lateral-control state of the art, reviewed brilliantly in 1937 in another joint paper with F. E. Weick.

Already in 1936 Jones had been probing 'far-out' ideas such as 'two-control' operation, involving lateral control through just a single control surface (replacing aileron and rudder). 'In general', he concludes, 'a reliable rolling-moment control that does not give a secondary adverse yawing moment would afford the most satisfactory means for two-control operation.' This seems to be prophetic of the Mitsubishi Aircraft Company's recent designs, with an airbrake-like spoiler being erected above the wing on the inside of the turn. These display the advantage foreseen by Jones and, additionally, make possible the use of full-span flaps for short-take-off purposes.

Good *handling* qualities required a combination of controllability with adequately low stick-force characteristics, which were progressively harder to achieve as aircraft increased in speed and size (powered controls being, of course, still in the very distant future for excellent reasons). Both ingenious mechanical linkages and aerodynamic devices were developed by Jones from 1936 onwards to reduce aileron operating force. This work continued as far as his important wartime reports on the use of a bevelled trailing edge to decrease the hinge moment of a control surface and so improve handling characteristics. At that time, also, his collaboration with Doris Cohen began with their joint paper pointing out the value of a stick-free stability analysis (stability of an aircraft with control surfaces free to move), and showing how it could be carried out and used.

During the 1941–5 war, too, he became involved with the Northrop tailless-aircraft projects which gave him great scope to use his fundamental knowledge of stability and controllability on aircraft configurations of a quite unconventional kind and gave him a first insight into some of the properties of sweepback. Certain guided weapons, again, which were essentially pilotless aircraft, proved ideal fields for the application of his deep understanding of the differential equations of aircraft dynamics and of their solutions under specified perturbing and controlling forces. The papers describing all this work make interesting reading.

In the meantime, however, Jones had begun fundamental researches of importance which went beyond the use of quasi-steady aerodynamic derivatives calculated from lifting-line theory. It had become necessary to take into account aircraft response at higher frequencies than such analysis could account for; response of the type which had been analysed on two-dimensional wing theory by Wagner in the time domain and by Theodorsen in the frequency domain. In 1938 Jones began, in the manner of his earlier work, with an aircraft dynamics based on convolution integrals, using the convenient but accurate representation

$$1-\tfrac{1}{6}e^{-0\cdot045s}-\tfrac{1}{3}e^{-0\cdot3s}$$

for the Wagner function (fractional shortfall of lift increment when the wing has travelled a distance of s half-chords with a new angle of attack). Two years later, however, he had already carried out a fundamental aerodynamic analysis of the corresponding effects for a wing of finite span.

Here, the influence of Max Munk's early teaching became both fully evident and explicitly acknowledged, as it was to be in so many of Jones's later papers. Admittedly, the main calculation followed the spirit of Wagner's analysis, with Wagner's vortex pairs (shed wake vortex plus its 'image' in the aerofoil) replaced by vortex rectangles (or, more strictly, a distribution of them corresponding to an elliptic load distribution); after intricate evaluations of integrals, Jones ended with approximate Wagner functions

$$0\cdot74-0\cdot27e^{-0\cdot38s} \quad \text{and} \quad 0\cdot60-0\cdot17e^{-0\cdot54s}$$

for aspect-ratios $A = 6$ and $A = 3$, respectively. The process of approximation, however, had involved a separate estimate of the value for $s = 0$, where the analysis by vortex rectangles was hard to apply for wings with a curved trailing edge. This is where Munk's ingenious method of application of added-mass theory gave particularly valuable results; essentially, the initial lift increment was the rate of change of cross-flow momentum, associated with the rate of change of cross-flow added mass as the wing chord was being initially extended through growth of the incipient vortex sheet. Here was the birth of the ideas leading to low-aspect-ratio wing theory.

In the meantime, with L. F. Fehlner, he applied the new theory to the effects of wings on tailplanes. This led for the first time to a tail lift induced by increased wing angle of attack which was initially positive (due to upwash behind the starting vortex) and then asymptoted to its normal negative value.

Jones never lost his interest in aircraft dynamics, and all his subsequent work on aircraft design for improved steady-state performance was influenced strongly by his understanding of factors important for stability and controllability. The exciting possibilities of flight at ever increasing Mach number began to draw him, however, in the early nineteen-forties, to a growing preoccupation with the steady-flow aerodynamic considerations that were fundamental to improving performance.

3. Steady-flow aerodynamics

In 1941 the collaboration with Doris Cohen that led later to their fine review *Aerodynamics of Wings at High Speeds* for the Princeton series moved forward with an elegant contribution to two-dimensional aerofoil theory. The classical conformal mapping $z + k^2z^{-1}$ is applied with *k small*, and with a choice of origin inside the aerofoil and close to a particular point on its surface, to effect a *small* modification of shape near that point whose effects on the velocity distribution (just multiplying it by $|1-k^2z^{-2}|^{-1}$) are readily calculated!

In the same year Jones began his attempts to improve the theory of steady flow over wings of finite span. His 1940 work using added mass had reminded him of Munk's discovery that for an elliptical flat plate there is a reduction in added mass per unit span for each section, below the value for an infinite plate of the same chord, by a constant factor $1/E$, where E is the elliptic integral giving the ratio of semiperimeter to span. He intuitively felt that a similar reduction must occur in the sectional lift coefficient. Although admitting in the later Princeton-series review that the reduction factor need not be and in fact is not identical, he performed a valuable service by his correct suggestion that a reduction of sectional lift–curve slope to a value in the general neighbourhood of $2\pi/E$ should apply even in inviscid theory. In other words, the reductions in sectional lift–curve slope below 2π commonly used in classical lifting-line theory must not be ascribed entirely to viscous effects but were about 50 % due to inviscid finite-span considerations. With induced downwash providing a reduction $\alpha_i = C_L/\pi A$ in effective angle of attack, the corrected lift coefficient

$$C_L = (2\pi/E)(\alpha - \alpha_i)$$

gave

$$C_L = 2\pi\alpha A/(EA+2),$$

in fairly close agreement with the Krienes accurate but highly intricate calculations for an elliptic wing which had just appeared.

The ideas in Jones's next two major papers on steady-flow aerodynamics matured over an extended period but their publication was held up for various reasons not wholly in his control until 1946 and 1947 respectively. The 1946 paper is celebrated as the first indication that steady-flow aerodynamics for certain wing configurations may take very closely related forms for both supersonic and subsonic flight. It is, of course, entitled 'Properties of low-aspect-ratio pointed wings at speeds below and above the speed of sound'.

Once the added-mass idea is fully mastered, the theory is simple indeed. It considers sections of the wing by successive planes perpendicular to the wing's motion through the air. The lift on such a section is the rate of increase of downward momentum for the air in contact with it. Thus, the lift per unit length is the forward speed V, times its component $V\alpha$ at right angles to the wing, times the gradient dm/dx of the added mass per unit length $m = \frac{1}{4}\pi b^2\rho$. (Here, b is the local span and it was not feasible for general wing shapes to correct the value of added mass per unit length given by two-dimensional theory.) The associated bound vorticity is elliptically distributed so that induced drag is a minimum.

However, any part of the wing behind the section of maximum span produces zero lift. This is because trailing vorticity with its elliptical distribution has constant downwash and so behaves like an infinite flat extension of the plate with uniform span and uniform added mass.

The total lift, then, is

$$L = \tfrac{1}{4}\pi b^2_{\max}\rho V^2\alpha, \quad \text{giving} \quad C_L = \tfrac{1}{2}\pi A\alpha.$$

The induced drag coefficient is

$$C_{Di} = C_L^2/\pi A = \tfrac{1}{2}C_L\alpha,$$

so that the direction of the resultant force bisects the vertical and the direction normal to the wing.

The paper already included the important idea that the approximation involved in, essentially, using solutions of the two-dimensional Laplace equation around each section would be as good for moderate supersonic speeds as for low speed; and would be even better when the Mach number M was close to 1. Tests of a pointed delta wing at $M = 1{\cdot}75$ showed good agreement with the theory up to an angle of attack of 1°, while at higher angles the influence of the quadratic term in C_L (about $2\alpha^2$) due to vortex force once leading-edge separation occurs, though obvious to us now in the data, could then plausibly be regarded as experimental scatter. Jones noticed, too, that the restriction to a pointed wing could be relaxed for subsonic wings, and that even an elliptical wing was described well at low speeds for $A < 1$ when the C_L given above was close to the exact values of Krienes.

An immediately associated paper, with K. Margolis, tackled the flow over slender bodies of revolution at supersonic speeds in a manner derived from Kármán's original work on that problem. Jones continued to take an interest in this field because of the importance of fuselage design for supersonic flight, and was to emphasize results on optimality of shapes like the Sears–Haack body of minimum wave drag for given enclosed volume. However, he did not pursue the aim of blending the slender-wing and slender-body theories achieved later by G. N. Ward and others, and his writings suggest no awareness of the fact, significant for slender delta wings, that their wave drag could be reduced to values less than that of the Sears–Haack optimal body of revolution with the same enclosed volume....

Even a man as many-sided as R. T. Jones may have one achievement that in retrospect can be seen to be clearly his greatest, and this surely was his NACA Report no. 863, 'Wing plan forms for high-speed flight', published in 1947. These Collected Works give a reader the advantage of studying the brief but cogent arguments for sweptback wings in that 5-page report in the context of the totality of work that led up to them, while showing also how the ideas in the Report were seminal for almost all its author's later contributions (including Dr Jones's major current interest: the yawed wing). The reader remembers, also, how the sweptback wing was practically as necessary as the jet engine to bring about the great revolution in air transport which, a decade later, began to bring peoples closer together than ever before through a vast increase in the volume, and decrease in the time, of international travel. To read, in the light of all that, these brilliantly written five pages is a rewarding experience. The discussion is continuously succinct and illuminating and no attempt at a reviewer's précis can be made!

The immediate follow-up to that work was a pair of important papers extending the sweepback theory to give a fuller treatment of the finite-span effects, respectively for supersonic and subsonic flow. The calculations are carried out elegantly by means of source distributions; soon, Busemann's conical fields were to be added to Jones's armoury for tackling these problems.

The supersonic data made it possible, next, for the author to do a classic series of first crude estimates for the lift–drag ratios of swept-wing aircraft at supersonic speeds. Instead of about $L/D = 6$ for a straight-wing design it would be reasonable to look for practically $L/D = 10$ in a swept-wing design. These results were further followed up in a 1948 paper emphasizing what they could mean for the range of a supersonic aircraft.

The next big theoretical advance concerned the boundary layer. Other writers noticed it simultaneously but Jones's 1947 paper gave the arguments in a particularly clear form. A yawed or swept wing of very high aspect-ratio has the boundary layer associated with its component of motion perpendicular to the leading edge uninfluenced by components of motion parallel to the leading edge because the velocity field has the same distribution in any plane perpendicular to the leading edge. This was a source of boundary-layer information of the highest value for the practical development of swept-wing aircraft.

Steady-flow aerodynamics was moving ahead supersonically now! Jones clarified in 1950 (again, simultaneously with others) the meaning of leading-edge singularities in thin-wing theory and the true significance of the theoretical leading-edge force for a real rounded leading edge. This understanding was most important for avoiding drags in excess of those predicted by theory.

Henceforth, reduction of induced drag and of wave drag became the central aim of Jones's theoretical work. To this end, countless new optima were calculated, and it is particularly valuable to have a compendium of them gathered together in this book.

The 1955 general article, 'Possibilities of efficient high-speed transport airplanes', was much influenced by all this research. Here we see the famous comparison (at low Mach number) of a two-dimensional aerofoil section (which had a lift–drag ratio of 300) with a circular wire of the same drag. The wire cross-section is practically invisible on the diagram because its diameter is so small compared even with the thickness (let alone the chord) of the aerofoil. This indicated how d'Alembert's so-called paradox (zero drag in potential flow) should really be regarded as a theorem, to the effect that 'good shapes' close to realizing potential flow have extremely low drag. The importance of minimizing components of drag absent from that comparison (induced drag for a finite-span wing and wave drag at supersonic speeds) becomes clear, and is the subject of much of the rest of the article. Further diagrams play an important part: the contrast between bodies of revolution of the same volume with minimum drag (assuming turbulent boundary layers) at low speed and at supersonic speeds, with their length/diameter ratios 2·5 and 15 respectively; and the contrast between aircraft models with the same zero-lift drag at supersonic speed, with the big swept-wing design dwarfing the little straight-wing model. Similar points were made the following year in the Prandtl memorial lecture.

One of the very important specialist papers on drag minimization reproduced here is the paper placing in a wider context the remarkable 'area rule' of Whitcomb, that aircraft wave drag at Mach numbers M near 1 depends on the longitudinal distribution of cross-sectional area. Jones showed how the limit of linear-theory drag as $M \downarrow 1$ is the Kármán drag of a body of revolution with the same area distribution. He also obtained for $M > 1$ a general linear-theory result of the same form, using the area distribution intercepted by parallel planes at the Mach angle, the final drag being an average of the different drags so calculated for all different orientations of the planes! This paper was influential towards promoting confidence in designs using 'waisted' fuselages suggested by area-rule considerations to obtain significant reductions in drag.

A few aspects of sharp-edged slender delta wings were, perhaps, ignored by Jones as noted already: the zero-lift drag potentially below the Sears–Haack minimum, and the accurately predictable and reproducible quadratic lift which is useful at supersonic speeds while allowing large lift coefficients to be obtained in highly stable flight at subsonic speeds. All of his thinking naturally favoured supersonic transport designs seeking optimality in both low-speed and supersonic flight through variable sweepback. The British and French, by contrast, pursued with success the great airworthiness advantage of an initial supersonic-transport design based on a fixed geometry with good flying qualities throughout the speed range. It required great confidence in the technical correctness of this solution to pursue it in the face of Boeing's competition with a variable-geometry project. Boeing's later announcement of reversion to a fixed-geometry slender delta was a happy moment for the Europeans, although the later decision by the U.S. government to cancel SST work was definitely regretted.

Now that second-generation supersonic transports are being widely studied, aerodynamicists are thrilled that Bob Jones, with NASA's active support, is pursuing with all his characteristic energy and insight the design which seems the logical consequence of all his life's work. This is the yawed wing which made its appearance in his original sweepback paper of 1947 and in several minimum-drag papers from 1951 onwards. Those of us who attended the International Congress of Aeronautical Sciences in Madrid (1958) were electrified by his emphasis on the high lift–drag ratios achievable with yawed elliptic wings; it is a pleasure to re-read here this paper containing so much that was new.

The first of the papers in this collection that give detailed proposals for a second-generation SST flying with a straight wing at low speeds, but pivoting the entire span of the wing so that it becomes yawed at high speeds, appeared in 1972. The performance advantages are fully stressed and then the author uses his aircraft dynamics knowledge together with experimental results on radio-controlled model aircraft to emphasize that lateral asymmetry does not rule out satisfactory flight dynamics.

It is fitting that the aeronautical section of this book ends with a series of papers on such an exciting forward-looking aeronautical project. In particular, an extended experimental investigation carried out jointly with L. A. Graham and F. W. Blotz is given in full detail. Here are the aerodynamic data for optimized combinations of fuselage and yawed wing which can be the basis of future designs in this area. Bill Sears states in his introduction: 'I, for one, fully expect to see future transport airplanes with "Jones oblique wings".' I may perhaps add that I, for another, concur!

M. J. Lighthill

The Finite Element Method in Partial Differential Equations. By A. R. MITCHELL and R. WAIT. Wiley, 1977. 198 pp. £6.95 or $13.50.

Finite Element Techniques for Fluid Flow. By J. J. CONNOR and C. A. BREBBIA. Newnes–Butterworths, 1976. 310 pp. £10.00.

The finite-element method has spawned a plethora of books in recent years, and these two will have to fight hard to find a niche on the bookshelf. The book by Mitchell and Wait is a textbook on the applied mathematics of the finite-element method, whereas Connor and Brebbia contribute a beginning text describing applications of the finite-element method to problems involving fluid flow. Both are suitable for senior-level or first-year graduate students. Neither is well suited to the research worker except as a basic introduction.

This review assumes an acquaintance with the finite-element method, and the uninitiated reader should begin with the excellent review by G. de Vahl Davis (*J. Fluid Mech.* **79**, 1977, 825–9) of the book *The Finite Element Method for Engineers* by Kenneth H. Huebner.

In the finite-element method the solution is expressed in terms of functions (usually polynomials) defined on a local basis, i.e. on the finite element. Mitchell and Wait emphasize the applied mathematics nature of their book by looking first at approximation to piecewise polymonials. Then variational principles are described, and the Galerkin method is introduced as applicable to a wider class of problems. The various choices of basis functions are described, including Hermite polynomials and the more generally used Lagrangian and serendipity basis functions. They do include the nine-node quadratic basis function, which is important in fluid mechanics (see below). The chapter lacks any comparison of advantages of the different basis functions, so the reader is left to make the choice. The methods for assembling and solving the element equations are omitted. A good chapter on the rate of convergence of some of the basis functions is included, and such information is useful if the reader is to choose a basis function. The next chapter treats time-dependent problems in a general way, describing how to include the time-dependent terms in the Galerkin method. The final chapter treats the patch test, an important test of basis functions, to see that the approximation will converge as the element sizes decrease. The last section is true to the thrust of the book: only 14 pages of applications, and those rather simple ones do not demonstrate the scope of complex geometries encompassed by the finite-element method. One of those applications is particularly germane; for the convective diffusion equation it shows how many nodes are necessary to avoid oscillations† and how to use a one-sided weighting function to dampen the oscillations while reducing the accuracy in the same manner as is achieved by artificial viscosity in finite-difference calculations. This point is essential if the finite-element method is to handle problems with strong convection because too many elements are otherwise required. The book is plainly oriented as a text in applied mathematics and for the beginner with those inclinations will serve admirably.

† The results quoted say that a solution derived using linear basis functions will oscillate unless a sufficient number of nodes are used but that the use of quadratic basis functions gives no oscillations. This result is contrary to experience and one of my students, Mr Ole Jensen, has shown that the middle nodes of a quadratic basis function do oscillate unless the number of nodes is sufficiently large. Mitchell and Wait only looked at the end nodes.

Connor and Brebbia introduce the subject by postponing the finite-element aspect and treating the solution method using trial functions defined over the whole domain, i.e. global basis functions. They introduce the method of weighted residuals and its sub-methods: Galerkin, collocation, least squares, moments, etc. A false impression is left that only the Galerkin method is applicable to nonlinear problems and that only the Galerkin method is used in conjunction with the finite elements. The introduction is clear, however, and will be useful to the beginning student. Finite elements are joined in the second chapter: the functional to be made stationary, the element equations, how those are assembled, and solved. Simple computer programs are given. Pedagogically this is sound because it gives the student something firm to grasp at the outset. Connor and Brebbia then treat the interpolating polynomials more clearly than Mitchell and Wait because of the simpler notation. The choice of basis functions is not as broad, however; the serendipity quadratic element is presented but the nine-node Lagrangian element is completely left out. The rate of convergence of the approximate solution to the exact solution is not discussed, and indeed the general question of errors in the approximate solution is ignored. The different basis functions are compared, and a recommended choice is the quadratic serendipity element.

Connor and Brebbia delve directly into fluid mechanics in chapter 4 by presenting the governing equations. Here the underpinnings are weak in places: the novice may not be aware that assumptions have been made. The fact that a stress tensor can be derived from the tractions is simply assumed and written down without discussion; the eddy viscosity is identified as dependent on the velocity field, with no warning to the beginner that assumptions are being made that must be tested experimentally; there are occasional errors in the equations. The chapter concisely summarizes the meaning of the Navier–Stokes equations without dwelling on the underpinnings.

The following chapters treat a succession of problems by first simplifying the equations then applying the finite-element method to them and describing applications. Some of the applications are extensive and were probably developed in sponsored research. All the applications are hampered by a tendency to accept the solution without finding out how well the equations are solved and hence which features are due to the problem and which ones are due to the solution method. Occasionally comparison with experiment is made. Seldom is the location of elements discussed, despite the fact that the accuracy may depend crucially on this choice. However the scope of possible applications involving complex geometries, free surfaces, and not-too-high velocities is demonstrated. The subjects treated include inviscid fluids, mainly shallow-water wave equations and the flow of inviscid fluid past bodies such as airfoils. No results of this latter application are given, however. Flow through porous media is considered for seepage problems around dams, for problems with free surfaces and in confined aquifers. This is clearly an area of expertise of the authors, but the application to petroleum reservoirs is not given. Each of the chapters has exercises, but these often deal with the physics of the subject rather than the finite-element aspects. Chapter 7 treats shallow-water circulation problems in detail with many field applications. Chapter 8 studies dispersion and handles the problem of strong convection by introducing a combined Eulerian–Lagrangian formulation; this is done in words, no equations are given, and the reviewer was unable to follow the application. This is an important area in which the finite-element method has difficulty, and is better treated by Mitchell and Wait. Indeed the book by Connor and Brebbia does not even warn the reader that oscillatory concentration profiles may develop if there is strong convection. The treatment of nonlinear diffusion is brief, and the iteration scheme is called Newton–Raphson, but is misnamed.

The final chapter concerns viscous incompressible fluids. There is a heavy bias towards using a stream function and vorticity, and the problems demonstrate the possible applications of the finite-element method: flow past an obstruction in a channel, flow past a cylinder, entry flow in a channel. The last calculation is described as 'most accurate away from the leading edge', and this is typical of all the applications: no discussion is given of the accuracy, or the effect of the size and distribution of the elements. For flow past a rectangle separation is achieved, and for high Reynolds numbers vortex shedding is demonstrated. These calculations are compared in gross features with finite-difference calculations, but the erros are not discussed. The pressure–velocity formulation is introduced briefly (no illustrations), and free boundaries are treated with the Lagrangian formulation. Again no illustrations are given, and the finite-element calculations of die swell of a Newtonian fluid flowing out of a jet (reported in this journal in 1974) are not even mentioned. When the velocity is represented by a quadratic trial function and the eight-node serendipity element is used, the pressures are unusual near singularities (they oscillate, for example). If a centre node is added to the trial function the pressure is accurately predicted. This information has been recently derived and illustrates the importance of new developments still occurring in the field. The delay in realizing this probably came about because the advocates of the finite-element method have been so eager to demonstrate the versatility of the technique that they have sometimes shown only gross results and failed to examine in detail the accuracy of different parts of the calculation. Clearly the field is mature enough now that true fluid mechanicians can employ the finite-element method, and improvements will come about as the limitations of the technique are revealed.

The book by Connor and Brebbia provides broad coverage suitable for the senior-level student but will be less useful to the research worker. Mitchell and Wait have provided an elementary treatment emphasizing the functional analysis aspects of the finite-element method which may be useful to both senior-level students and research workers. Fluid mechanics is superficial in one book and non-existent in the other; but perhaps the application of the finite-element method to fluid mechanics is moving too fast to expect more at present.

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